A derived variable physical photon velocity explains the apparent accelerated universe without invoking dark energy

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Abstract

A variable $c(t)$ has been previously proposed as an alternative to dark energy as the cause for the apparent acceleration of the universe found with supernovae Ia on high red-shift galaxies, but without an analytic expression for $c(t)$. We have herein derived a variable photon velocity from the assumption of anisotropic and homogeneous universe which is consistent with being the physical photon velocity $c(t)$ in that it uses physical clocks and rulers. The derivation makes $c(t) = \alpha E$, the cosmic scale factor times the Hubble ratio in normalized variables. But a variable $c(t)$ no longer keeps the Lorentz transform invariant. Instead, locally we have a generalized Lorentz transform and generalized Minkowski metric. By introducing a generalized time for which the differential is $c(t)dt$, we create a covariant derivative and a Minkowski metric in order to use Riemann algebra applied to generalized field equations. With this we find that the Hubble ratio has the same dependence on the cosmic scale factor as it does for a constant photon velocity.

Measurements of the fine structure in clouds of cosmic matter experimentally demonstrate that some physical "constants" do change with cosmic time. We use dependences on $c(t)$ of various physical "constants" which keep a normalized quantum Lagrangian invariant. These explain the apparent acceleration of the universe found with supernovae Ia on high red-shift galaxies by assuming a flat universe

with no dark (vacuum) energy, and by experimentally determining the supernovae Ia luminosity to be proportional to $1/c(t)$.

keywords: cosmology theory—distances and redshifts—elementary particles—ISM lines and bands—relativity—supernovae Ia

1 Introduction

Measurements of supernovae Ia at high z [1][2][3] have been interpreted as evidence for the existence of dark energy, which has led to its wide spread acceptance by the cosmology community, even though there is not a real physical understanding of it. There have been suggestions in the literature [4][5][6] that a variable photon velocity might also explain the experimental observations. However, there has not previously been a good analytic expression for such a variable photon velocity. This paper reports on a derivation of such an expression from the usual assumptions of a homogeneous and isotropic universe and the definition and invariance of the space-time line element.

The assumption that the universe is homogeneous and isotropic does not automatically lead to a constant photon velocity. In fact, the Robertson-Walker (RW) space-time line element[8][9], which can be derived from the assumption of this symmetry, allows for a variable photon velocity $c(t)$ which depends on the cosmic time t. Any coordinate system that results from a diffeomorphic covariant transform from the RW coordinates will describe the same physical system as does the RW coordinates. These constitute a geometric manifold with a variable photon velocity. In particular if the coordinates from one such transform can be read on physical rulers and clocks and is variable, then the photon velocity described by its coordinates will be physical, and all other diffeomorphic covariant transforms, including the original RW coordinates, will also describe physical systems with the same physical variable photon velocity. We have discovered a group of coordinate systems which have the same variable, observer-independent, photon velocity $v_0(t)$ at their origin, which, if $v_0(t) = c(t)$, we will show have physical clocks and rulers. These "physical origin" coordinate systems are a subset of all covariant transforms from an RW system (t, χ, θ, ϕ) constrained by the requirement that their spatial points move at a negative velocity when measured on the RW radial coordinate and their time $T(t, \chi, \theta, \phi)$ be measured on clocks attached to the spatial coordinates at a distance $R(t, \chi, \theta, \phi)$ from the origin. (In this paper "origin" will always mean spatial origin, where, generally, the observer is assumed to be located).

But a variable photon velocity has far reaching implications. A large number of other physical "constants" depend on it, so they presumably also change with cosmic time. The measured change in the fine structure spectra of heavy metals at high z demonstrates that such a change does take place [6]. In addition a variable $c(t)$ no longer keeps the Lorentz transform invariant, so that the equivalency principle and Einstein's field equations must be modified. Instead, locally we must have a generalized Lorentz transform and generalized Minkowski metric. Following the suggestion of Magueijo [7], we introduce a generalized time $\hat{t}$ for which $d\hat{t} = c(t)dt$ in order to define a covariant derivative and create a Minkowski metric for use in a generalized field equation. Of course, the generalized Lorentz transform must reduce to the normal Lorentz transform for events whose time separation is small compared to cosmic time in order that all the laws of atomic, nuclear, and particle physics remain valid. Also, the generalized field equation must reduce to Einstein's field equation for the measurements in the Milky Way or other close-in galaxies which have been used to confirm Einstein's equation.

Using Magueijo [7] as a guide, we determine the dependence of other physical "constants" on $c(t)$ by making the quantum Lagrangian invariant to $c(t)$. This combination explains an apparent accelerated universe which we will show agrees with observation of Supernovae Ia at high z.

We proceed by using the RW coordinates with $c = c(t)$ applied to a radial world line to find the radial 4-velocity and 4-acceleration of a point on the frame of the covariantly transformed coordinate system in terms of both the RW coordinates and the transformed coordinates themselves (Sect 2). This will yield partial differential equations (PDEs) for the allowable covariant coordinate transformations from the RW to the transformed system (Sect 3). We integrate these PDEs for those transformed coordinates which have a diagonal metric (Sect 4). These all have the same variable photon velocity $v_0(t)$ at the origin, which, if $v_0(t) = c(t)$, makes $c(t) = \alpha E$ (Sect 5). Isotropy requires all of the transforms to be a newly defined generalized Lorentz transforms close to their origin (Sect 6). We develop criteria for physicality and show that there exists a group of transforms, both diagonal and non-diagonal, which meet these criteria at the origin with the same $v_0(t)$ as the diagonal coordinates (Sect 7). By introducing the generalized time $\hat{t}$, a generalized field equation is defined, which locally reduces to the generalized Minkowski metric and Lorentz transform (Sect 8). This allows us to calculate $a(t)$ and thus $c(t)$ (Sect 9). The dependence of other physical "constants" on $c(t)$ is derived from an invariant normalized quantum Lagrangian (Sect 10), which then explains the observed apparent universe acceleration (Sect 11). We discuss feasibility of earth experiments to mea-

sure $c(t)$ in Sect 12. In Sect 13 we point out that $c(t) = v_0(t)$ does not remove the horizon problem even though $c(t)$ goes to infinity as t goes to zero.

2 The transformation from RW coordinates

For a homogeneous and isotropic universe, one can derive without the use of relativity the RW moving coordinate system whose metric in polar coordinates (r, θ, ϕ) [10, page 412] (adapted to $c(t)$) is

$$ds^2 = c(t)^2 dt^2 - a(t)^2 [d\chi^2 + r^2 d\omega^2], \tag{1}$$

where t is the universal cosmic time, χ is the co-moving curvilinear radial spatial variable ($\chi > 0$), and $a(t)$ is the scale factor of the RW metric, $d\omega^2 \equiv d\theta^2 + sin^2\theta d\phi^2$, and

$$r = \begin{cases} \sin\chi, \ k = 1, \\ \chi, \ k = 0, \\ \sinh\chi, \ k = -1, \end{cases} \tag{2}$$

where k is the RW spatial curvature variable to indicate a closed, flat, or open universe, resp.. In order to define a covariant derivative [7], it will be convenient to introduce the time related quantity $\hat{t}$ defined by

$$\hat{t} \equiv \int_0^t c(t) dt, \tag{3}$$

so that the line element becomes

$$ds^2 = d\hat{t}^2 - a^2 (d\chi^2 + r^2 d\omega^2). \tag{4}$$

Since this is the exact form of the original derivation of the RW metric, it is clear that introducing $c(t)$ does not inviolate the derivation. We will take the origin of the transformed radius to be the same as the origin of the RW radius: $R = 0$ at $\chi = 0$. We will require that $T = t$ at $\chi = 0$, since the time on clocks attached to every galactic point is t, including the origin. We will also make the magnitude of the photon velocity in the (T, R) system to be unity at the origin at the present time $T_0 = t_0$. This makes the unit of $c(t)$ be the value of what is measured by an observer at the origin at the present time (e.g., on earth now).

We will assume that the angular coordinates θ_s and ϕ_s of the new transformed coordinates are given by the angular coordinates of the RW system.

We will be initially confining our attention to only radial trajectories, so that the angular coordinates are fixed, and T and R will be functions only of t and χ. Thus

$$\begin{array}{l} T = T(t,\chi),\ \ R = R(t,\chi) \\ \theta_s = \theta,\ \ \phi_s = \phi. \end{array} \tag{5}$$

The transform of the differentials is given by

$$\begin{aligned} &dT = T_t dt + T_\chi d\chi,\ \ dR = R_t dt + R_\chi d\chi, \\ &d\theta_s = d\theta = 0,\ \ d\phi_s = d\phi = 0, \end{aligned} \tag{6}$$

where the subscripts (other than s) indicate partial derivatives with respect to the subscript variable. We will henceforth drop the subscript s.

We will assume a general form for the metric expressed in transformed coordinates with spherical symmetry. [10, page 335]:

$$ds^2 = A^2 c(t)^2 dT^2 - B^2 dR^2 - 2Cc(t)dRdT - a^2r^2d\omega^2. \tag{7}$$

where B and C are implicit functions of (T, R) and explicit functions of (t, χ). We assume that $A \equiv 1$, which is equivalent to an assumption that the clocks on the transformed system are attached to the spatial coordinates. Also, since both the RW metric and our transformed metric are spherically symmetric, the coefficients of the angular variables are the same.

We will use as the RW coordinates $(\hat{t}, \chi, \theta, \phi)$ and as the transformed coordinates $(\hat{T}, R, \theta, \phi)$, where $d\hat{T} = cdT$, so that the transformed metric becomes

$$ds^2 = d\hat{T}^2 - B^2 dR^2 - 2CdRd\hat{T} - a^2r^2d\omega^2, \tag{8}$$

and the transform from $(\hat{t}, \chi)$ to $(\hat{T}, R)$ becomes

$$\begin{aligned} d\hat{T} &= T_t d\hat{t} + cT_\chi d\chi, \\ dR &= \tfrac{1}{c} R_t d\hat{t} + R_\chi d\chi, \end{aligned} \tag{9}$$

Since the contravariant velocity vector of a point on the transformed system in the transformed coordinates is $(1, 0, 0, 0)$, this will transform from the vector U^μ in the RW coordinates $(\gamma, \frac{\gamma\hat{V}}{a}, 0, 0)$ the same as $d\hat{T}, dR$ in eq 9:

$$\begin{array}{rcl} 1 &=& T_t\gamma + \frac{1}{a}cT_\chi\gamma\hat{V}, \\ 0 &=& \frac{1}{c}R_t\gamma + \frac{1}{a}R_\chi\gamma\hat{V}. \end{array}, \tag{10}$$

where $\hat{V}$ and γ are defined as

$$\hat{V} \equiv a(t)\left(\frac{\partial \chi}{\partial \hat{t}}\right)_R, \ \gamma \equiv \frac{1}{\sqrt{1-\hat{V}^2}}. \tag{11}$$

$\hat{V}$ is related to the ordinary RW radial velocity V of a point on the transformed frame by

$$V = a(t)\left(\frac{\partial \chi}{\partial t}\right)_R = c(t)\hat{V}, \tag{12}$$

since V is the RW differential distance $a(t)d\chi$ divided by the RW differential time dt, taken for the co-located point R on the transformed frame.

Manipulating the second line of eq 10 gives

$$\hat{V} = -\frac{\hat{a}R_t}{R_\chi}, \tag{13}$$

where

$$\hat{a}(t) = \frac{a(t)}{c(t)}. \tag{14}$$

Thus $\hat{a}(t)$ has the dimensions of time, and $a(t)$ has the dimensions of length. (We use various related non-physical quantities with hats to reduce the proliferation of $c's$).

If we invert eq 9, put this into the RW metric (eq 4), and use eq 10, we verify that A is 1, and that the expressions for B^2 and C in the metric (eq 7) become:

$$\begin{aligned} B^2 &= (\gamma T_t \tfrac{a}{R_\chi})^2[1-(\tfrac{T_\chi}{\hat{a}T_t})^2], \\ C &= \gamma^2 T_t \tfrac{a}{R_\chi}[\hat{V} + \tfrac{T_\chi}{\hat{a}T_t}]. \end{aligned} \tag{15}$$

We will consider the contravariant acceleration vector of a test particle moving with the same velocity as a point on the transformed frame. This covariant derivative of the velocity vector can be expressed in both the transformed coordinates

$$\begin{aligned} \hat{A}^{\hat{T}} &= \tfrac{d\hat{U}^{\hat{T}}}{ds} + \Gamma^T_{\hat{T}\hat{T}} U^{\hat{T}}\hat{U}^{\hat{T}} = g^* + \tfrac{CC_{\hat{T}}}{B^2+C^2}, \\ \hat{A}^R &= \tfrac{d\hat{U}^R}{ds} + \Gamma^R_{\hat{T}\hat{T}} U^{\hat{T}}\hat{U}^{\hat{T}} = g + \tfrac{C_{\hat{T}}}{B^2+C^2}, \end{aligned} \tag{16}$$

and the RW coordinates

$$\begin{aligned} \hat{A}^{\hat{t}} &= \tfrac{d\hat{U}^0}{ds} + \Gamma^0_{11}U^1\hat{U}^1 = \gamma\left(\tfrac{\partial\gamma}{\partial\hat{t}}\right)_R + a\dot{a}\tfrac{\gamma^2\hat{V}^2}{a^2} = \gamma^4\hat{V}\left(\tfrac{\partial\hat{V}}{\partial\hat{t}}\right)_R + \tfrac{\dot{a}}{a}\gamma^2\hat{V}^2, \\ \hat{A}^\chi &= \tfrac{d\hat{U}^1}{ds} + \Gamma^1_{01}(U^0\hat{U}^1 + \hat{U}^0U^1) = \gamma\left(\tfrac{\partial(\gamma\hat{V}/a)}{\partial\hat{t}}\right)_R + 2\tfrac{\dot{a}}{a}\tfrac{\gamma^2\hat{V}}{a} = \gamma^4\tfrac{1}{a}\left(\tfrac{\partial\hat{V}}{\partial\hat{t}}\right)_R + \tfrac{\dot{a}}{a^2}\gamma^2\hat{V}, \end{aligned} \tag{17}$$

where $C_{\hat{T}} = \left(\frac{\partial C}{\partial \hat{T}}\right)_R$, the Γs are the affine connections for their respective coordinate systems, $\hat{U}$ and $\hat{A}$ are the velocity vector and acceleration vector of the test particle, the dot represents differentiation by $d\hat{t}$, $g \equiv d^2R/d\hat{T}^2 = d^2R/dT^2$ is the ordinary acceleration for the test particle, and $g^* \equiv d\hat{U}^{\hat{T}}/d\hat{T}$.

It will transform like $d\hat{T}, dR$ (eq 9):

$$\begin{aligned} \hat{A}^{\hat{T}} &= \hat{A}^t T_t + c\hat{A}^\chi T_\chi, \\ \hat{A}^R &= \tfrac{1}{c}\hat{A}^t R_t + \hat{A}^\chi R_\chi. \end{aligned} \tag{18}$$

We can obtain a relation between g, g^*, and C by setting to zero the scalar product of the covariant velocity vector and the contravariant acceleration vector in transformed coordinates:

$$g_{\mu\nu}\hat{U}^\mu \hat{A}^\nu = g^* + \frac{CC_{\hat{T}}}{B^2 + C^2} - C(g + \frac{C_{\hat{T}}}{B^2 + C^2}) = 0 \tag{19}$$

so that

$$g^* = Cg. \tag{20}$$

If we insert the value of g^* from eq 20 and C from eq 15 into eqs 16 and 17 and rewrite the two equations of 18 in terms of their factors, using eq 13, we obtain

$$\begin{aligned} gF_1 + \tfrac{C_{\hat{T}}}{B^2+C^2}F_1 &= \tfrac{R_\chi}{a}F_1F_2, \\ g + \tfrac{C_{\hat{T}}}{B^2+C^2} &= \tfrac{R_\chi}{a}F_2, \end{aligned} \tag{21}$$

where the factors are defined as

$$\begin{aligned} F_1 &\equiv \hat{V} + \tfrac{T_\chi}{\dot{a}T_t}, \\ F_2 &\equiv \gamma^2 \left(\tfrac{\partial \hat{V}}{\partial \hat{t}}\right)_R + \tfrac{\dot{a}}{a}\hat{V}. \end{aligned} \tag{22}$$

3 The partial differential equations for a diagonal transform

We are interested in seeing whether there is a diagonal solution ($C = 0$) of eq 21, and if there is, what are the PDEs that determine its dependent variables. (In this paper we will use the adjective "diagonal" to describe any quantity associated with a metric which is diagonal when expressed in transformed coordinates, such as diagonal solution, diagonal metric, diagonal transform,

diagonal coordinates, etc.). We can see from eq 15 that when $F_1 = 0$, then $C = 0$, so there is diagonal solution of eq 21 for $F_1 = 0$, $g = \frac{R_\chi}{a} F_2$.

We will now find a differential equation with only $\hat{V}$ as the dependent variable. Transposing $F_1 = 0$ yields another expression for $\hat{V}$:

$$\hat{V} = -\frac{T_\chi}{\hat{a} T_t}. \tag{23}$$

Putting eq 23 into eq 10 produces a simple relation for T_t:

$$T_t = \gamma. \tag{24}$$

Now we can write a formula for T, using eq 23 and eq 24:

$$T = t + \int_0^\chi T_\chi \partial \chi_t = t + \int_0^\chi (-\hat{a}\gamma \hat{V}) \partial \chi_t, \tag{25}$$

where the symbol $\partial \chi_t$ signifies that the integral is to be carried out with respect to χ with t held constant, and we have used the boundary condition that at $\chi = 0$, $T = t$. Eq 25 can be partially differentiated with respect to t (giving γ) and then with respect to χ and use eq 11 to obtain a PDE for $\hat{V}$:

$$\hat{V}_t + \hat{V}_\chi \left(\frac{\partial \chi}{\partial t} \right)_R = \left(\frac{\partial \hat{V}}{\partial t} \right)_R = -\frac{1}{\hat{a}} \frac{d\hat{a}}{dt} \hat{V}(1 - \hat{V}^2). \tag{26}$$

We will find these diagonal transforms very useful since they represent the simplest covariant transform. We will be able to find a general solution of the PDE's for them, including a formula for calculating the variable photon velocity at the origin $v_0(t)$. This is very convenient, since we will find that there is a physical origin group of coordinates, diagonal and non-diagonal, all of which have this same $v_0(t)$ (see Sect 7).

4 General solution of the partial differential equations for a diagonal metric

Thus, we shall now try to find expressions for the transformed coordinates with a diagonal metric. The PDEs we need to solve for $T(t, \chi)$, $R(t, \chi)$, and $\hat{V}(t, \chi)$ are the three equations 13, 23, and 26 subject to the boundary conditions at $\chi = 0$ of $R = 0, T = t, \hat{V} = 0$, and the photon velocity equal to one at $t = t_0, \chi = 0$. Eq 26 can be rewritten as

$$\frac{\partial \hat{V}_R}{\hat{V}(1 - \hat{V}^2)} = -\frac{\partial \hat{a}_R}{\hat{a}}, \tag{27}$$

where the subscript on the partial differential indicates the variable to be held constant. This can be integrated with an integration constant $\ln\kappa$ ($\kappa > 0$). Since the integration is done at constant R, then $\kappa = \kappa(R)$, and inversely, $R = R(\kappa)$. Integrating eq 27, we get

$$\hat{V} = -\frac{\kappa}{\sqrt{\hat{a}^2 + \kappa^2}}, \tag{28}$$

where we have selected the minus sign for the square root in accordance with our assumption that the transformed frame have a negative velocity when measured in RW coordinates.

At this point, R is an unknown function of κ. The various possible coordinate systems which solve our PDEs are characterized, in large part, by the function $R(\kappa)$. But for all, in order for $\hat{V}$ to vanish when $R = 0$, κ must also; so always

$$\kappa(0) = 0. \tag{29}$$

One observation we can make without any more work is that, as long as $\kappa(R)$ remains finite, $\hat{V}$ goes to -1, and V goes to $-c(t)$ (see eq 12), for $\hat{a}(t) = 0$, i.e. for $t = 0$, the horizon. This is different from any of the three definitions of distance assuming a constant photon velocity at the origin treated by Fletcher (including the proper distance) [11], for all of which $\hat{V}$ became -1 at a finite t instead of at $t = 0$.

Let us now look at lines of constant $\kappa(R)$, i.e. constant R, in t, χ space. Eq 11 can be integrated for χ with use of eq 28 at constant κ to give the following general solution, for an arbitrary function $f(\kappa)$ and an arbitrary upper limit t^*:

$$\chi(t, \kappa) = f(\kappa) + \int_t^{t^*} \frac{\kappa \partial s_\kappa}{\hat{a}(s)\sqrt{\hat{a}^2(s) + \kappa^2}}. \tag{30}$$

$f(\kappa)$ is the value of $\chi(t, \kappa)$ for $t = t^*$. For an open universe, we expect that if R is kept constant the galactic point χ that will be passing any given R will eventually approach zero as RW time t approaches infinity. Thus for this case we can set $f(\kappa) = 0$ at $t^* = \infty$.

For a closed universe, $\hat{a}$ will be periodic, so that $\hat{V}$ goes to zero at some point, which we will take to be t^*, after which we may choose a positive sign for $\hat{V}$ to represent a collapsing universe. (Actually, for this derived $c(t)$, if we solve the field equation in Sect 8, we will find $t^* = \infty$, so the universe never collapses). In order for χ to remain finite at that point and still give physical coordinates at the origin (see Sect 7.1), we will make

$$f(\kappa) = M \int_0^{t^*} \frac{\kappa \partial s_\kappa}{\hat{a}(s)\sqrt{\hat{a}^2(s) + \kappa^2}}. \tag{31}$$

Here, the constant M will be a function of the universe energy density, so that it will become zero at the transition between a closed and a flat universe.

At this point, we have obtained $\hat{V} = \hat{V}^*(t, \kappa)$ from eq 28 and have also obtained the function $\chi(t, \kappa)$. We can in principle invert eq 30 to obtain κ in terms of t and χ: $\kappa = K(t, \chi)$. This gives us the velocity function $\hat{V}(t, \chi) = \hat{V}^*(t, K(t, \chi)$. If the function $R(\kappa)$ were known, we would then also have $R(t, \chi) = R(K(t, \chi))$.

$T(t, \chi)$ can be found by noting from eqs 23, 24, and 28 that

$$T_\chi = -\hat{V}\hat{a}T_t = -\hat{V}\hat{a}/\sqrt{1 - \hat{V}^2} = \kappa. \tag{32}$$

By substituting eq 32 into eq 25, and integrating over κ instead of χ by dividing the integrand by the partial of eq 30 with respect to κ, we find an expression for $T(t, \chi)$. For an open universe it can be written as

$$T(t, \chi) = t + \int_t^\infty \left[1 - \frac{\hat{a}(s)}{\sqrt{\hat{a}^2(s) + \kappa^2}}\right] \partial s_\kappa, \tag{33}$$

where κ is put equal to $K(t, \chi)$ after integration at constant κ in order to get $T(t, \chi)$. For a closed universe we can use eq 31 to get $f(\kappa)$ and thus $T(t, \chi)$:

$$T(t, \chi) = t + M \int_0^{t^*} \left[1 - \frac{\hat{a}(s)}{\sqrt{\hat{a}^2(s) + \kappa^2}}\right] \partial s_\kappa + \int_t^{t^*} \left[1 - \frac{\hat{a}(s)}{\sqrt{\hat{a}^2(s) + \kappa^2}}\right] \partial s_\kappa. \tag{34}$$

This completes the solution. These are all the possible solutions for the diagonal case. We note that these are in reality all the same system except for different calibrations of the R axis with $R = R(\kappa)$.

5 Photon velocity close to the origin for a diagonal metric

We would like to find an expression for photon velocity in the transformed diagonal coordinate system at the origin, that is as κ approaches zero by eq 29. Using eqs 13, 23, and 24 inserted into 15, the metric (eq 7) becomes

$$ds^2 = c^2 dT^2 - \left(\frac{a\gamma}{R_\chi}\right)^2 dR^2 - a^2 r^2 d\omega^2. \tag{35}$$

Although $B(t,\chi)$ has been determined from radial trajectories, when γ and R_χ are expressed in terms of t and χ, the metric of eq 35 is valid for all trajectories, since for spherical symmetry, B cannot depend on the angular coordinates. The radial photon velocity is given by dR/dT for $ds = 0$:

$$v_p = \frac{R_\chi}{\gamma \hat{a}}. \tag{36}$$

(For $C = 0$ we write the positive value of the photon velocity, but understand that in all cases for a diagonal metric there are both positive and negative values).

From eqs 36 , we get

$$v_p(t,\chi) = \frac{1}{\gamma \hat{a}} \frac{dR(\kappa)}{d\kappa} \left(\frac{\partial K}{\partial \chi}\right)_t. \tag{37}$$

K_χ can be written in an inverted form by taking the derivative of eq 30 with respect to κ:

$$\left[\left(\frac{\partial K}{\partial \chi}\right)_t\right]^{-1} = \left(\frac{\partial \chi}{\partial \kappa}\right)_t = f'(\kappa) + \int_t^{t^*} \frac{\hat{a}(s)\partial s_\kappa}{(\hat{a}^2(s) + \kappa^2)^{3/2}}. \tag{38}$$

We will let $v_0(t)$ be the photon velocity for a diagonal metric at the origin. To find it, we set $\kappa = 0$ in eq 38, $\gamma = 1$, and use eq 37 to give

$$\frac{1}{v_0(t)} = \hat{a}\kappa'(0) \left[f'(0) + \int_t^{t^*} \frac{ds}{\hat{a}^2(s)}\right]. \tag{39}$$

$\kappa'(0)$ is a constant to be adjusted to make $v_0(t_0) = 1$. It should be noted that the value of $v_0(t)$ is independent of the functional form of $\kappa(R)$, and is therefor the same for all diagonal transforms with the same $\hat{a}(t)$.

We will be showing (sect 7) that it is consistent to have the physical photon velocity $c(t)$ equal $v_0(t)$. Under this assumption, eq 39 with a change in the integration variable from t to $a(\hat{t})$ eq 39 becomes

$$\frac{1}{a} = \kappa'(0) \left[f'(o) + \int_a^{a^*} \frac{c(a)da}{a^2 \dot{a}}\right], \tag{40}$$

remembering that the dot indicates differentiation by $\hat{t}$. By differentiation of both sides of eq 40 by a, we can obtain

$$c(t) = \frac{1}{\kappa'(0)} \dot{a} = \alpha E, \tag{41}$$

where α is the normalized scale factor

$$\alpha \equiv a/a_0, \tag{42}$$

and E is the normalized Hubble ratio $H(t)$

$$E \equiv \frac{H}{H_0} \equiv \frac{1}{H_0}\frac{\dot{a}}{a}. \tag{43}$$

(The subscript 0 denotes the value at $t = t_0$, the present time; $\kappa'(0) = a_0 H_0$). From this relation, the generalized field equation (sect 8) will enable us to evaluate α and E and thus $c(t)$.

6 Transformed metric near the origin

Since V goes to zero as R goes to zero, and we are assuming isotropy of the universe, the photon velocities in all directions become the same, with C going to zero at the origin for all the transformed coordinates.

The metric for all the transformed coordinates for small R can thus be written

$$ds^2 = c^2 dT^2 - \frac{c^2}{v_p(t,0)^2}[dR^2 + R^2 d\omega^2], \tag{44}$$

where $v_p(t,0)$ is the variable photon velocity at the origin, which is given by $v_0(t)$ in eq 39 for diagonal metrics, but may possibly be different for some non-diagonal metrics.

In this limit, since $V(t,0) = 0$ and $C(t,0) = 0 = \frac{T_\chi}{\hat{a}T_t}(t,0)$ and $T_t(t,0) = 1$ (eq 10), then from, eq 15

$$\frac{1}{v_p(t,0)} = \frac{1}{c}B(t,0) = \frac{\hat{a}(t)}{R_\chi(t,0)} = \frac{\hat{a}(t)\chi}{R}, \tag{45}$$

where the last equality has used the first terms of the series expansion of R around $\chi = 0$ $[R_t(t,0) = 0]$. Therefor, all transformed coordinates near the origin become

$$\begin{aligned} T &= t, \\ R &= v_p(t,0)\hat{a}(t)\chi. \end{aligned} \tag{46}$$

Note that the value of R at $T = t_0$ is given near the origin by $a(t_0)\chi$ (since $v_p(t_0,0) = 1, \hat{a}(t_0) = a(t_0)$), the expression for which all measurements of distance reduce for small χ [Weinberg 1972 page 424].

If $v_p(t,0) = c(t)$, we can call eq 44 a generalized Minkowski metric because for a range of times, $\delta T << t$, the metric becomes the Minkowski metric. It can be written with the generalized time $\hat{T}$:

$$ds^2 = d\hat{T}^2 - [dR^2 + R^2 d\omega^2], \tag{47}$$

It is the metric for a generalized Lorentz transformation between a frame (dT', dR') moving at a radial velocity v relative to a second frame (dT, dR):

$$\begin{aligned} dT' &= \gamma'(t)(dT - \tfrac{v}{c(t)^2} dR), \\ dR' &= \gamma'(t)(-v dT + dR), \end{aligned} \tag{48}$$

where $\gamma'(t) = 1/\sqrt{1 - v^2/c(t)^2}$. Of course, because of spherical symmetry, this generalized Lorentz transform applies to a frame moving in any direction for differential distances measured in the direction of the velocity v.

7 The Physicality Criteria

We would like to find those transformed coordinates which are physical since this is the type of coordinate system with which our physical laws are formulated. The cosmological principle gives us a unique opportunity to do this, because it says that the clocks and rulers on every galactic point, when properly calibrated and synchronized, are the same. So, to be physical we need the transform to use the same clocks and rulers and to have the same photon velocity as the co-located galactic point. This latter requirement arises because at every galactic point we have the generalized Minkowski metric for which the photon velocity is independent of the observer, including co-located observers on the transformed frame moving with a velocity V. For criteria of sameness, we apply a version of the equivalency principle that the laws of physics are the same in every physical coordinate system. We assume measurements in every physical coordinate system made on a clock stationary in that system are all the same, and measurements on a ruler for constant time in that system are all the same, for a universal line increment ds.

We are already assured that the clocks which measure T are the same as the clocks at the origin by making the metric coefficient of $d\hat{T}$ be unity, since then $c(t)dT(t,R) = ds$ at constant R, and $c(t)dt(t,\chi) = ds$ at constant χ, so that the attached clocks on the transformed coordinates are the same as the clocks at every galactic point. From the co-located galactic point having earth rulers, we get a physicality requirement on rulers in the radial

direction, by making the transformed radial distance ∂R_T at a constant time T on local transformed clocks be identical to the radial distance $a(t)\partial\chi_t$ (eq 14) on a co-located galactic point taken at a constant time t on the local galactic clocks. This must be true for a given differential of proper distance ds, so the radial rulers will be the same if

$$\left(\frac{\partial R}{\partial s}\right)_T = a(t)\left(\frac{\partial \chi}{\partial s}\right)_t. \tag{49}$$

By putting the left side of eq 49 into eq 7 and the right side into eq 1, we obtain the radial ruler requirement $B = 1$, or if $\hat{B} \equiv B/c$, then

$$\hat{B}(t,R) = \frac{1}{c(t)}. \tag{50}$$

This must be independent of R since the clocks, rulers, and photon velocities on co-located galaxies are all the same at the same cosmic time t, independent of R.

Since $\hat{B}$ and the photon velocity are generally a function of R, none of our transformed coordinates meet the physicality requirement for all R except for the zero density universe (see Sect 9). Another exception is at the origin, where we will find a subset of the transformed coordinates that can meet the physicality criteria for a small range of R.

7.1 Physical origin for a diagonal metric

We can make the transformed coordinates physical for a limited range of R near the origin of a diagonal metric by requiring that the partial derivative of $\hat{B}(t,R)$ re R vanish at the origin. This satisfies both the ruler requirement and the constant photon velocity requirement, since $v_p = 1/\hat{B}$ for a diagonal metric. Eqs 35 and 38 show that

$$\hat{B}(t,R) = \frac{\hat{a}\gamma}{R_\chi} = \hat{a}\gamma\left[f_R - \int_t^{t^*} \hat{V}_R(t',R)\frac{\partial t'_R}{\hat{a}(t')}\right]. \tag{51}$$

By partial differentiation, we obtain

$$\hat{B}_R = \frac{\hat{a}\gamma_R}{R_\chi} + \hat{a}\gamma\left[f_{RR} - \int_t^{t^*} \hat{V}_{RR}(t',R)\frac{\partial t'_R}{\hat{a}(t')}\right]. \tag{52}$$

Since $\gamma_R(t,0) = 0$, $\hat{V}_{RR}(t,0) = 0$ will make $f_{RR}(0)$ (eq 31) and the square bracket of eq 52 and thus $\hat{B}_R(t,0)$ vanish at the origin. If we partially

differentiate eq 28 twice, we see that $\hat{V}_{RR}(t,0) = 0$ requires that $\kappa_{RR}(0) = 0$. Thus, those diagonal transforms which have R linear in κ for small R have physical clocks and rulers which measure the physical photon velocity near the origin if

$$c(t) = 1/\hat{B}(t,0) = v_0(t) \tag{53}$$

given by eq 39. This makes $c(t) = \alpha E$ (eq 41).

These criteria are consistent with the more general criteria developed by Bernal et al[15]. In their notation for x_1 and x_2 translated to the notation in this paper, if we assume the RW metric has the length measure $dx_1 = a(t)d\chi$ and the transformed length measure is $dx_2 = dR$, then at the origin $\partial_2 x_1 = a(t)\chi_R = a(t)/R_\chi = c(t)/v_0(t)$, and $\partial_1 x_2 = R_\chi/a(t) = v_0(t)/c(t)$. Since these two partial derivatives are equal at the origin when $v_0(t) = c(t)$, the two systems meet the Bernal criterion for using the same rulers there. A similar relation applies to clocks in the two systems, which are the same in both systems for all R.

If $v_0(t)$ is to be considered physical, every other transform that meets these same physicality requirements at its origin must have the same $v_0(t) = c(t)$. There are, of course, many coordinate systems, including transforms from the RW system, which do not have physical coordinates anywhere. In order to find another transform which is physical at the origin, we will try to find a physical origin for a non-diagonal transform from eq 1 in an open universe.

7.2 Physical origin for a non-diagonal metric in an open universe

For a non-diagonal metric, the criterion at the origin for constant rulers requires that $\hat{B}_R = 0$. We will introduce the symbol

$$\Upsilon \equiv -\frac{T_\chi}{\hat{a}T_t}, \tag{54}$$

so that $\hat{B}$ in eq 15, usng T_t from eq 10, becomes

$$\hat{B}(t,R) = \frac{\hat{a}}{R_\chi}\frac{\sqrt{1-\Upsilon^2}}{1-\hat{V}\Upsilon}. \tag{55}$$

Eq 11 is still valid for non-diagonal metrics, so for an open universe

$$\chi = -\int_t^\infty \hat{V}(t',R)\frac{\partial t'_R}{\hat{a}(t')}, \tag{56}$$

and

$$\frac{1}{R_\chi} = \left(\frac{\partial \chi}{\partial R}\right)_t = -\int_t^\infty \hat{V}_R(t', R)\frac{\partial t'_R}{\hat{a}(t')}. \tag{57}$$

By partial differentiation of $\hat{B}$ (eq 55), noting that $V(t, 0) = 0$ and since $C(t, 0) = 0 = \Upsilon(t, 0) = 0$, we get

$$\hat{B}_R(t, 0) = \hat{a}(t)\left(\frac{\partial(1/R_\chi}{\partial R}\right)_t (t, 0) = -\hat{a}(t)\int_t^{t^*} \hat{V}_{RR}(t', 0)\frac{dt'}{\hat{a}(t')} \tag{58}$$

Thus, one general requirement for making an open metric, either diagonal or non-diagonal, have radial rulers equal galactic rulers near the origin is to set $\hat{V}_{RR}(t, 0) = 0$; i.e., have $\hat{V} \propto R$ to first order in R.

The constant photon velocity imposes the additional condition $C_R(t, 0) = 0$, which is equivalent to $\Upsilon_R(t, 0) = \hat{V}_R(t, 0)$ (eq 15). Even though this requirement makes the metric diagonal to first order in R, there is no a priori reason that it have the same $v_0(t)$ as the diagonal metric, although we will find that in fact it has.

We need first to find an expression for the transformed time for a non-diagonal metric. From eq 10 we obtain

$$\left(\frac{\partial T}{\partial t}\right)_R = T_t + T_\chi \frac{\hat{V}}{\hat{a}(t)} = \frac{1}{\gamma(t, R)}. \tag{59}$$

Since eq 59 is valid for both diagonal and non-diagonal coordinates, the integration constant is obtained by requiring T be the diagonal expression (eq 34) when $\gamma(s, R)$ equals $\sqrt{1 + \kappa^2/a^2}$. Thus the integral of eq 59 becomes

$$T \equiv G(t, R) = \int_0^t \frac{\partial t'_R}{\gamma(t', R)} + \int_0^\infty \left[1 - 1/\gamma(t', R)\right] \partial t'_R. \tag{60}$$

Note that $G(t, 0) = t$ since $\gamma(t, 0) = 1$. If we partially differentiate eq 60, we get $G_R(t, 0) = 0$ and partially differentiate a second time we obtain

$$G_{RR}(t, 0) = \int_t^\infty \hat{V}_R^2(t', 0)\partial t'_R. \tag{61}$$

To find an expression for Υ, we note from eq 10 that $T_t = 1/\gamma(1 - \hat{V}\Upsilon)$ and by the chain rule that $T_\chi = G_R R_\chi$. Inserting these into eq 54, after some manipulation we get:

$$\Upsilon = \frac{-1}{-\hat{V} + \hat{a}/\gamma R_\chi G_R}. \tag{62}$$

The partial derivative of eq 62 at the origin then becomes

$$\Upsilon_R(t,0) = -\frac{G_{RR}(t,0)}{\hat{a}(t)/R_\chi(t,0)} \tag{63}$$

We can obtain a solution by making the integrands of G_{RR} (eq 61) and $1/R_\chi$ (eq 57) be proportional with a proportionality constant of $\kappa'(0)$. This makes

$$\hat{V}_R(t,0) = -\frac{\kappa'(0)}{\hat{a}(t)}, \tag{64}$$

and $G_{RR}(t,0)R_\chi(t,0) = \kappa'(0)$. Then from eq 63 we get

$$\Upsilon_R(t,0) = -\frac{\kappa'(0)}{\hat{a}(t)} = \hat{V}_R(t,0), \tag{65}$$

showing that eq 64 is a solution. Thus the physical origin non-diagonal photon velocity in an open universe becomes the same $v_0(t)$ as the diagonal coordinates (use eq 64 in eqs 57 and 47), which it must if it is to be considered equal to $c(t)$. It seems likely that this is also true for a closed universe.

It should be observed that up to this point in the paper, we have not used the field equation. The RW coordinates, the transform from them, the variable photon velocity at the origin, and the physicality criteria depend only on the assumed symmetry of homogeneity and isotropy of the universe and the invariance of the space-time line element ds^2.

7.3 Non physicality of the RW proper coordinates

The customary concept of radial cosmic distance is the RW proper distance, defined as $a(t)\chi$, which for constant χ goes to zero as t and $a(t)$ go to zero. If we use this distance as part of the RW proper coordinate system, $R^* = a(t)\chi,\ T^* = t$, the metric of eq 1 using eq 14 becomes

$$ds^2 = c^2 dT^{*2} - \left[dR^* - \frac{R^*}{a}\frac{da}{dt}dT^*\right]^2 - \frac{r^2}{\chi^2}R^{*2}d\omega^2. \tag{66}$$

This system is the reverse of our transformed coordinates in that the ruler physicality requirement is met for all distances but the clock physicality requirement is not met at any distance, except at the origin. The coefficient A^{*2} of dT^{*2} is given by

$$A^{*2} = c^2 - \frac{R^{*2}}{a^2}(\frac{da}{dt})^2 \tag{67}$$

At the origin, $A^{*2} = c^2$ to first order in R^*, so both the clocks and the rulers meet the physicality requirement at the origin.

The radial proper coordinate photon velocity

$$v_p^*(t, R^*) = \pm c(t) + \frac{R^*}{a}\frac{da}{dt}. \tag{68}$$

At the origin $v_p^*(t, 0) = c(t)$ and the metric becomes the generalized Minkowski (since near $R^* = 0, \chi = r$). However, $v_p^*(t, 0)$ is not constant to first order in $\hat{R}$. Thus $v_p^*(t, 0)$ does not meet our physicality requirement for constant photon velocity. The coordinate photon velocity becomes the physical photon velocity at a point on the R^* axis, the origin, because of its intrinsic composition, not because it is derived by its equivalence to a constant galactic photon velocity. c(t) is not determined by $v_p^*(t, 0)$, and may assume any function of t.

8 Revised field equation for a variable photon velocity.

If a variable photon velocity $c(t)$ is a physical reality, we need to revise the Equivalency Principle and the field equation to reflect the expectation that all non gravitational physical measurements, anywhere and any time, obey the generalized Lorentz transforms (eq 48) with the generalized Minkowski metric (eq 47) rather than the usual Lorentz transforms and the usual Minkowski metric. The use of $\hat{t}$ or $\hat{T}$ in the field equation accomplishes this:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{69}$$

where $G_{\mu\nu}$ is the Einstein contracted curvature tensor determined from the coefficients $g_{\mu\nu}$ multiplying the differentials in the metric of eq 4, or of eq 8, Λ is the cosmological "constant" possibly representing some kind of vacuum energy density, G is the gravitational "constant" and $T_{\mu\nu}$ is the stress-energy tensor. Locally, this will give the generalized Minkowski metric (eq 47) and generalized Lorentz transform (eq 48) for dT and dR close to the origin, as well as for dt and $\hat{a}d\chi$.

It will be convenient to use the variables $\hat{t}, r, \theta, \phi$ in the generalized RW metric (eq 4):

$$ds^2 = d\hat{t}^2 - a^2(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 cos^2\theta d\phi^2). \tag{70}$$

so the $g_{\mu\nu}$ of eq 69 are the coefficients of eq 70, exactly the same as the usual RW metric with constant $c = 1$ in the variables t, r, θ, ϕ. We will make the usual assumption that the universe is an ideal fluid with an energy density of ρc^2 and pressure p, so that we can write the two significant field equations [12, page 729] for $a(\hat{t})$ as

$$\frac{3\dot{a}^2}{a^2} + \frac{3k}{a^2} - \Lambda = \frac{8\pi G}{c^2}\rho, \tag{71}$$

and

$$+2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \Lambda = -\frac{8\pi G}{c^4}p, \tag{72}$$

where the dots represent derivatives with respect to $\hat{t}$.

Now, the first equation can be multiplied by $a^3/3$, differentiated, and subtracted from $\dot{a}a^2$ times the second to give

$$\frac{d}{d\hat{t}}\left(\frac{G\rho}{c^2}a^3\right) = -\frac{3G}{c^4}\dot{a}a^2 p - \frac{a^3}{8\pi}\dot{\Lambda}, \tag{73}$$

where we have allowed for the possibility that G and Λ may be functions of $c(t)$. For small p and $\dot{\Lambda}$,

$$G\rho a^3/c^2 = G_0\rho_0 a_0^3/c_0^2, \tag{74}$$

a constant. If the energy density consists of n particles per unit volume of mass m, so $\rho = nm$, then the conservation of particles requires na^3 be constant. This makes

$$\frac{Gm}{c^2} = constant. \tag{75}$$

Even when p and $\dot{\Lambda}$ are not negligible as in the early universe, it is reasonable to assume that matter particles are conserved well into the relativistic particle era and that the relation of constants in eq 75 will continue, so that for matter density eq 74 remains valid. But for radiation, we assume $kT \propto 1/a$ as it is for $c = 1$ so that radiation energy density is proportional to $1/a^4$ (see sect 11).

9 Calculation of $a(t)$ and $c(t)$.

Following Peebles [13, page 312], we define

$$\Omega \equiv \frac{8\pi G_0\rho_0}{3H_0^2 c_0^2} \tag{76}$$

and

$$\Omega_r \equiv \frac{-k}{H_0^2 a_0^2} \tag{77}$$

and

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}. \tag{78}$$

If $kT \propto 1/a$ as it is for $c = 1$, then we can introduce a radiation term:

$$\Omega_R = \frac{8\pi G_0}{3H_0^2 c_0^4} B_0 (T_0 a_0)^4 \tag{79}$$

where B_0 is the Stefan-Boltzmann's constant divided by c and multiplied by 1.68 to account for neutrino energy density [13, page 164]. Eq 97 will show that B_0 is independent of $c(t)$ so that $\Omega_R \approx 1.7x10^{-4}$, a constant. Radiation makes an important contribution to $\dot{a}$ only when $\alpha < \Omega_R/\Omega$. The Ωs are defined so that

$$\Omega_R + \Omega + \Omega_r + \Omega_\Lambda = 1. \tag{80}$$

Then the normalized Hubble ratio E in eq 43 is determined by eq 71:

$$E = \sqrt{\frac{\Omega_R}{\alpha^4} + \frac{\Omega}{\alpha^3} + \frac{\Omega_r}{\alpha^2} + \Omega_\Lambda}. \tag{81}$$

which allows us to evaluate $c(t) = \alpha E$ in eq 41. At $t = t_0$: $\alpha_0 = 1$, $E(\alpha_0) = 1$, and $c_0 = 1$.

The physical time t becomes

$$c_0 H_0 t = \int_0^\alpha \frac{d\alpha}{c\alpha E} = \int_0^\alpha \frac{d\alpha}{\alpha^2 E^2}. \tag{82}$$

where $c_0 = 1$ is written explicitly for clarity. For $\Omega = 1$: $c = \alpha^{-1/2} = (t_0/t)^{1/4}$, and $c_0 H_0 t = \alpha^2/2$, whence $c_0 H_0 t_0 = 1/2$, $\hat{a}/a_0 = (t/t_0)^{3/4}$.

It is interesting to consider the limiting case of a zero density universe ($\Omega = 0, \Omega_r = 1/a_0^2 H_0^2$). Eq 41 makes $c = 1, H_0 a_0 = 1$ and eq 82 makes $\alpha = H_0 t, \hat{a} = a = t$. Eq 30 gives $\chi = csch^{-1}(t/\kappa)$, or $K(t,\chi) = tsinh\chi$. We can then find from eqs 28 and 11 that $V(t,\chi) = -\tanh\chi$ and $\gamma = \cosh\chi$ so that the photon velocity for all distances by using eq 37:

$$v_p = \frac{dR}{d\kappa}\frac{K_\chi}{\gamma t} = \frac{dR}{d\kappa} \tag{83}$$

Thus the physicality condition is met for all R with $R = K$ and $v_p = v_0 = c(t) = 1$, so that the complete transform with eq 34 becomes

$$\begin{aligned} R &= t\sinh\chi, \\ T &= t\cosh\chi. \end{aligned} \tag{84}$$

This solution has been known ever since Robertson [14] showed that this transformation from the RW co-moving coordinates at zero density obeyed the Minkowski metric. What is new is that this solution was derived from the equations we obtained for our covariant diagonal transforms and our physicality requirement applied for all distances.

For experiments attempting to measure the variation of the photon velocity at the origin at the present time, the derivative of $c(t)$ (eq 41) will be more useful:

$$\frac{1}{c_0 H_0}\left[\frac{1}{c}\frac{dc}{dt}\right]_{t=t_0} = 1 - \frac{3}{2}\Omega - \Omega_r = -\frac{\Omega}{2} + \Omega_\Lambda. \tag{85}$$

Notice that this fraction is negative when matter dominates, and increases in magnitude from zero at zero density to $-1/2$ at the critical universe density. A vacuum energy density opposes the gravitational effect of matter, and as it increases, it causes the variation to go to zero before reversing it from a decreasing function of time to an increasing function.

10 Other physical constants.

First, we should recognize that the spectra of atoms and molecules can depend on $c(t)$. Thus, the fine structure constant α_f in SI units [16] is

$$\alpha_f = \frac{e^2}{4\pi\epsilon_0\hbar c}, \tag{86}$$

and the Rydberg constant expressed as a frequency is

$$R_\infty = \alpha_f^2 \frac{m_e c^2}{4\pi\hbar} = \frac{e^4 m_e}{\epsilon_0^2 (4\pi\hbar)^3} \tag{87}$$

which both can depend on $c(t)$ since if $c(t)$ depends on time t, we must allow for the possibility that ϵ_0 will also depend on it because $c(t)^2 = 1/\epsilon_0(t)\mu_0(t)$. [Note that the $4\pi\epsilon_0$ is often omitted in the fine structure constant since it is unity in Gaussian coordinates, but it is essential here if we are to consider a variable $c(t)$].

Recently, observers have measured a shift in the fine structure in absorption spectra of matter between distant quasars and us [6]. For a range of cosmic times, they attribute this to a change in α_f for which they found $\Delta\alpha_f/\alpha_f = -0.72 \pm 0.18x10^{-5}$ over the red shift range $0.5 < z < 3.5$. This experimentally demonstrates that some physical "constants," must be

changing with cosmic time. However, the magnitude is far too small to be caused solely by the $c(t)$ calculated here. This implies that if $c(t) = v_0(t)$, some other physical "constants" must vary as well, such as those found in eqs 69, 86 and 87.

A variable physical photon velocity has been previously investigated for general consistency with all our physical laws [7] with suggestions for how this might explain many cosmological puzzles [4][5]. We are here concerned with a derived variable physical photon velocity and not with assumed variations to explain these puzzles. We here propose a possible variation of other physical "constants." Following the suggestions of Magueijo [7], we can obtain two relations: (1) To make the interactive term of a normalized quantum Lagrangian independent of $c(t)$ for particles with a charge g_i in a field ($g_i = e$ for electromagnetic field), we make $g_i/\hbar c$ constant; (2) To make the matter term independent of $c(t)$, we make $m_i c^2/\hbar c$ constant, where m_i is the mass of a particle of i matter. We can put these relations together with eq 75 and the experimental observation that α_f is almost constant to obtain a parameterized dependence on c^q:

$$g_i, \hbar c, m_i c^2, \frac{c^4}{G}, \epsilon_0, \frac{1}{\mu_0 c^2} \propto c^q. \tag{88}$$

It is possible that Magueijo's minimal coupling requirement $q = 0$ applies, for which the rest energy of individual particles is independent of $c(t)$. This will make the quantities in eq 88 all constant. Unless otherwise stated in this paper, we will take $q = 0$. If we apply eq 88 to an emitted spectral line ν_{nm} (eq 87), we get

$$\nu_{nm} \propto R_\infty \propto c(t), \tag{89}$$

independent of the parameter q. The small change in the separation of spectral frequencies determining an apparent shift in α_f with $c(t)$ in intergalactic gas could then be considered a secondary effect as the ratio of α_f to R_∞ decreases with $c(t)$.

If we review the evidence for unchanging constants [12, sect 38.6], the arguments for no change with spatial location is not violated by the present determination that they change only with cosmic time t. All the arguments for no change with time seem to be related only to the fine structure constant α_f, which we are also assuming doesn't change with time.

The change in $\nu_{nm}(c)$ will change the red shift formula [10, page 416, using $\hat{a}$]. Although the elapsed time Δt_e and the photon period are each increased by $\hat{a}_0/\hat{a}_e$ by the expansion of the universe, the frequency starts

out at $c(t_e)$ times the normal frequency at the origin so that

$$1 + z = \frac{\hat{a}_0}{c(t_e)\hat{a}_e} = \frac{a_0}{a_e} = \frac{1}{\alpha_e}, \tag{90}$$

where t_e is the cosmic time of emission. For $\Omega = 1$ this will make $c(t) = \sqrt{1+z}$.

11 Distance determination from supernovae Ia

The computed distance d_L [10, page 421] from a standard candle is derived from the received power/area being reduced from the emitted power/area. Because of the expansion of the universe, the photon freqency is reduced by $\hat{a}_e/a_0$ and the time beween the emission of photons is increased by the same ratio. Distance. including space curvature, further reduces the power per unit area by $4\pi(a_0 r_e)^2$. Planck's constant increases the energy of each photon by c_e to give a net ratio of received power/area to emitted luminosity of $(\hat{a}_e/a_0)^2 c(t_e)/(4\pi a_0^2 r_e^2)$. This results in an apparent distance given by

$$d_L = \frac{a_0^2 r_e}{c_e^{1/2}\hat{a}_e}(\frac{L_s}{L_e})^{1/2} = a_0(1+z)c_e^{1/2} r_e (\frac{L_s}{L_e})^{1/2}, \tag{91}$$

where we have allowed for the possibility that the emitted luminosity L_e of the standard candle might change compared to its value L_s determined in local galaxies at the present time.

If we expand c_e and r_e in powers of z, we get

$$\begin{aligned} c_e &= \alpha E = 1 + \tfrac{z}{2}(\Omega - 2\Omega_\Lambda) + O(z^3) \\ r_e &= sinh\left[\int_{t_e}^{t_0} \tfrac{cdt}{a}\right] = \tfrac{1}{a_0 H_0}\left[z - \tfrac{z^2}{2}(1 + \tfrac{1}{2}\Omega - \Omega_\Lambda) + O(z^3)\right]. \end{aligned} \tag{92}$$

In order to allow for variation in L_e, we will parameterize it by making it proportional to $1+z)^n$:

$$L_e/L_s = (1+z)^n. \tag{93}$$

d_L thus becomes to second order in z

$$d_L = \frac{1}{H_0}\left[z + \frac{z^2}{2}(-n + \Omega + \Omega_r + \Omega_\Lambda) + O(z^3)\right]. \tag{94}$$

High z measurements of supernovae Type Ia [1] [2] [3] indicate an apparent accelerating universe. They have shown that the best fit to their data

using a constant photon velocity is obtained for a flat universe ($\Omega_r = 0$) by $\Omega = .3 \pm .15, \Omega_\Lambda = .7 \mp .15$(68.3% confidence). For a constant $c = 1$ the power series expansion of $d_L^*(z)$ becomes

$$d_L^* = \frac{1}{H_0}\left[z + \frac{z^2}{2}(1 - \frac{1}{2}\Omega + \Omega_\Lambda) + O(z^3)\right]. \tag{95}$$

This gives a coefficient of z^2 of $0.78 \pm .11$. Using eq 94 with $c(t)$, we get the z^2 coefficient to be 0.75 with $\Omega = 1, n = -1/2$ ($\Omega_\Lambda = \Omega_r = 0$). That is, it appears that we can explain the observed anomalous acceleration by the variable photon velocity calculated for a flat universe without requiring the existence of vacuum energy. However, it requires the maximum emission of the supernovae Type Ia to be proportional to $1/(1+z)^{1/2} = 1/c(t)$. It should not be surprising that the peak luminosity of the SNIa varies with $c(t)$. Thus, for a hydrogen-like atom, the emitted power also varies with $c(t)$, but in this instance being proportional to $c(t)$. For the SNIa it will take a theoretical analysis of the physics using $c(t)$ to see whether it agrees with the experimentally determined $1/c(t)$.

Recently, the Supernovae Ia have been extended to even higher z[17]. For high z, the observed spectrum shifts to longer wavelengths by a factor of $1+z$ compared to the spectrum at low z (by definition of z). As experiments reach to these higher values of z, comparison of the derived d_L with experiments is a little subtle. Thus, the correction to the rest frame of the SNIa means that the measured times should be divided by $\hat{a}_0/\hat{a}_e = E$ instead of $1+z$ used in the paper. For $\Omega = 1$ this correction is $E = (1+z)^{3/2}$. Also, we should use the full expression for d_L in eq 91 instead of a power series in z (eq 94). For $\Omega = 1$ and $n = -1/2$ this becomes

$$H_0 d_L = 2(1+z)(-1 + \sqrt{1+z}) \tag{96}$$

In addition, the spectrum in the rest frame of SNIa increases in frequency by $c(t_e)$ compared to the low z spectrum, so that the wavelengths in the rest frame are the observed wavelengths divided by $E/c(t_e) = 1+z$, just as assumed in the Tonry paper[17]. Preliminary reworking their measurements with these adjustments indicate that they agree with eq 96 within the quoted uncertainties.

The success of this comparison suggests future work with $c(t)$, especially where vacuum energy has been previously invoked, e.g., CMB, gravitational lensing, and dynamical estimates of galactic cluster masses.

Another conclusion we can draw from $q = 0$ is confirmation that kT for decoupled radiation varies inversely with $a(t)$ (and not with $\hat{a}(t)$). Thus, if

the radiation starts out as a Planck distribution of photons with frequency, each frequency will be decreased by $1/\hat{a}$. so that the distribution is maintained as the universe expands (as is observed) with the maximum of the distribution occuring at ν_{max}, then $kT \propto h\nu_{max} \propto 1/c\hat{a} = 1/a$. There is no separate dependence on $c(t)$ of k or T; the determination of the units of the temperature scale determines k. It should be observed that when we consider radiation in the early universe that the value of B_0 in eq 79 is

$$B_0 = 1.68 \frac{\pi^2}{60} \frac{k^4}{(\bar{h}c)^3}, \tag{97}$$

a constant for $q = 0$, so Ω_R is also constant, as was assumed in the formula for $E(\hat{a})$(eq 81).

It would also be a test of the theory if the secondary effect of the fine structure spectrum in the heavy metal atoms at high z could be calculated and compared to observation. Another challenge is to calculate the age of old globular clusters using $c(t)$ with $\Omega = 1$ to see whether it is shortened enough to be significantly less than the 16% shorter age of the universe, $1/(2H_0)$ instead of $2/(3H_0)$.

12 Feasibility of confirmation by earth based experiments.

A natural question is whether the predicted variability of a physical $c(t)$ is, in principle at least, subject to confirmation by earth based experiments. For a critical universe density with $H_0 = 50$ km/s/Mpc and no vacuum energy, so that $c(t) = (t_o/t)^{1/4}$, we would have a fractional decrease in the measured photon velocity of $\Delta t/4t_0 = .8x10^{-14}$ in 10^4 secs. We note, however, that if we have vacuum energy density, that this will affect the photon velocity in the opposite way, so it could cause an increase in the photon velocity (Eq 85), rather than a decrease.

Recent developments project that fractional frequency measurement accuracies of spectral emissions should achieve 10^{-17} over a period of 10^4 secs in the near future [Berquist et al 2001; Kleppner 2001]], which is a thousand times more accurate than the predicted cosmic variation of $c(t)$. If our proposed variation of physical "constants" described in the previous section is valid, then $\nu \propto c(t)$ and $\lambda = c(t)/\nu =$ constant, so that measuring the change in ν would be equivalent to measuring the change in $c(t)$.

13 Horizon of CMB

We would like to see if the increasing $c(t)$ when t goes to zero allows communication between CMB coming to us from different directions. The CMB is characterized by the microwave frequency reaching us has a red shift of about $1 + z = 1/\alpha_e = 1000$. In the time from the beginning of the universe, photons could transverse laterally (constant χ) a distance Δ of

$$\Delta = \int_0^{t_e} c(t)dt = \frac{1}{H_0}\int_0^{\alpha_e} \frac{d\alpha}{\alpha E} \tag{98}$$

Along a transverse angle θ_e in the observer's coordinate system, this distance is given by $\Delta = a_e r(\chi_e)\theta_e$ for small angles, where $r(\chi)$ is given by eq 2, and χ_e is given by

$$\chi_e = \int_{t_e}^{t_0} \frac{cdt}{a} = \frac{1}{a_0 H_0}\int_{\alpha_e}^{1} \frac{d\alpha}{\alpha^2 E} \tag{99}$$

For a flat universe, where $r(\chi) = \chi$, the angle of the lateral horizon is given by

$$\theta_e = \frac{\int_0^{\alpha_e} \frac{d\alpha}{\alpha E}}{\alpha_e \int_{\alpha_e}^{1} \frac{d\alpha}{\alpha^2 E}} \tag{100}$$

For $\Omega = 1$, $\alpha_e >> \Omega_R$ and $\theta_e = \alpha_e^{1/2}/3 = .01$ radians. Only for angles smaller than this are the CMB ripples coordinated (except by inflation). Inflation is still needed to establish communications in the early universe.

A rather surprising result is that this horizon is exactly the same as for a constant $c = 1$. In that case, $d\hat{t} = cdt$ becomes dt and $\hat{a}(t)$ becomes $a(t)$ so the equation eq 100 is the same.

14 Conclusions

Using the assumption of a homogeneous and isotropic universe and the invariance of the space-time line element, we have derived a group of "physical origin" coordinates, covariantly transformed from the RW coordinates, which at their origin are consistent with being physical with a generalized Minkowski metric and an observer-independent variable photon velocity $v_0(t)$, the same for the whole group, which includes metrics that are both diagonal and non-diagonal. By consistent, we mean that if their photon velocity equals the physical photon velocity and is constant to first order in R at their origin, then they use the same rulers and clocks that we have at

every galactic point. We have here explored the possibility of their being equal, which determines $c(t)$ to equal the cosmic scale factor times the Hubble ratio: $c(t) = \alpha E$ in normalized variables. We have also shown that the coordinate photon velocity of the RW proper coordinates is not constant to first order in R at the origin and does not determine the physical photon velocity $c(t)$.

Assuming we know this physical photon velocity $c(t)$, we must alter the Equivalency Principle and Field Equation so that locally there is a generalized Minkowski metric and a generalized Lorentz transform. We propose to alter the field equations by allowing c, G, and Λ to be functions of the cosmic time t and by introducing a generalized time $\hat{t}$ for which $d\hat{t} = c(t)dt$, which yields the required generalization. This alteration shows that the equation for determining $a(\hat{t})$ with a variable $c(t)$ is the same as $a(t)$ with $c = 1$. We have demonstrated that in the limit of a zero density universe the physical origin coordinates meet our physicality criteria for all distances with a constant photon velocity and become the well known coordinates first transformed from RW by Robinson[14].

The kinematics on distant galactic material should be calculated using a co-moving generalized Minkowski metric, whose photon velocity is $c(t)$. Measurements of the fine structure in the absorption spectra from distant quasars caused by intervening clouds of matter show a change from that measured on earth. These measurements experimentally confirm that physical "constants" do change with cosmic time. The measurements show a very small fractional change, far too small to be caused solely by the $c(t)$ calculated in this paper. This implies that if we have a variable photon velocity, some other physical "constants" should vary as well. We have followed Magueijo's approach[7] for a possible dependence of "constants" on $c(t)$ in Sect 10, which indicate that the formulas for red shift and distance should be modified. The proposed modification explains the apparent universe acceleration found with supernovae Ia on high red-shift galaxies by assuming $c(t) = v_0(t)$ in a flat universe with no vacuum energy, and with the experimentally determined requirement that the SNIa luminosity be proportional to $1/c(t)$. However, it does not eliminate the horizon problem.

The agreement with the measurements of the Supernovae Ia suggest that these variable "constants" also be applied to other astrophysical observations where vacuum energy has been previously invoked, e.g., CMB, gravitational lensing, and dynamical estimates of cluster masses. It would also be a test of the theory if the secondary effect of the fine structure spectrum in the heavy metal atoms at high z could be calculated and compared to observation. Another challenge is to calculate the age of old globular clusters with $c(t)$

to see whether it is shortened enough to be still significantly less than the 16% shorter age of a critical universe, $1/2H_0$ instead of $2/3H_0$.

This variation of $c(t)$ should in principle be measurable by earth experiments. For a critical universe density of matter this fractional decrease should be about 10^{-14} in 10^4 secs, which should be measurable if the projected fractional frequency accuracy of 10^{-17} in 10^4 secs is achieved, and if the frequency is proportional to $c(t)$ as proposed in section 10. If the energy density due to the cosmological constant is dominant, the photon velocity and frequency will increase with time instead of decreasing, providing another test of the existence or non-existence of dark energy.

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